

SOLUTIONS to H/w #3.

- (1.) The probability to be in a degenerate state with energy E_s will be proportional to:

$$P(E_s) \propto d(E_s) g_R(U_0 - E_s),$$

where the degeneracy $d(E_s)$ is the number of compatible microstates of the system and $g_R(U_0 - E_s)$ is that of the reservoir.

Now I'll repeat the standard procedure of magically converting the unknown function $g_R(U_0 - E_s)$ into an exponential:

$$\ln g_R(U_0 - E_s) - \bar{g}_R(U_0 - E_s) = \bar{g}_R(U_0) - E_s \left(\frac{\partial \bar{g}_R}{\partial U} \right)_{V,N} (U_0) + O(E_s^2)$$

We can ignore the $O(E_s^2)$ term, presumably because E_s is infinitely small compared to U_0 — the energy of an infinitely large reservoir.

Now, by definition, $\frac{1}{\tau} = \left(\frac{\partial \bar{g}_R}{\partial U} \right)_{V,N} (U_{eq})$, where U_{eq} is the equilibrium value of the reservoir energy, i.e. $U_{eq} = U_0 - E_{eq}$. Again, because the reservoir is huge, the difference between $\frac{1}{\tau} = \left(\frac{\partial \bar{g}_R}{\partial U} \right)_{V,N} (U_0 - E_{eq})$ and $\left(\frac{\partial \bar{g}_R}{\partial U} \right)_{V,N} (U_0)$ is negligibly small compared to either of these quantities so finally,

$$\ln g_R(U_0 - E_s) = \text{const} - E_s \frac{1}{\tau}, \text{ so}$$

$$g_R(U_0 - E_s) = \text{const } e^{-E_s/\tau}$$

(*) Then $P(E_s) = \text{const } d(E_s) e^{-E_s/\tau}$

But we must have $\sum_{E_s} P(E_s) = 1 = \text{const} \sum_s d(E_s) e^{-E_s/\tau}$

Hence, $\frac{1}{\text{const}} \stackrel{\text{def}}{=} Z = \sum_{E_s} d(E_s) e^{-E_s/\tau}$

One can see that this definition of the partition function is completely